where

$$\begin{aligned} d_1(\psi) &= \Delta_y\psi - \Delta_x\psi, \\ d_2(\psi) &= |\nabla_y\psi|^2 - |\nabla_x\psi|^2. \end{aligned}$$

Then, we obtain

$$\begin{aligned} J_1 &= -2s\,\mathrm{Im}\int_\Omega z\partial_t(\nabla_y\varphi)\cdot\nabla_y\overline{z}dxdydt + 2s\,\mathrm{Im}\int_\Omega z\partial_t(\nabla_x\varphi)\cdot\nabla_x\overline{z}dxdydt \\ &= -2s\,\mathrm{Im}\int_\Omega \left(-2\gamma^2\beta t\right) z\varphi\nabla_y\psi\cdot\nabla_y\overline{z}dxdydt \\ &\quad +2s\,\mathrm{Im}\int_\Omega \left(-2\gamma^2\beta t\right) z\varphi\nabla_x\psi\cdot\nabla_x\overline{z}dxdydt \end{aligned} \tag{3.22}$$

and

$$\begin{aligned} J_2 &= \sum_{i,j=1}^m \mathrm{Re}\int_\Omega 4s\varphi_{y_iy_j}z_{y_j}\overline{z}_{y_i}dxdydt - \mathrm{Re}\sum_{j=1}^m\sum_{i=1}^n\int_\Omega 4sz_{y_i}\overline{z}_{x_j}\varphi_{y_ix_j}dxdydt \\ &= \sum_{i,j=1}^m \mathrm{Re}\int_\Omega 4s\gamma\varphi\left(\psi_{y_iy_j}+\gamma\psi_{y_i}\psi_{y_j}\right)z_{y_j}\overline{z}_{y_i}dxdydt + \int_\Omega 4s\gamma^2\varphi\left|\nabla_y\psi\cdot\nabla_y z\right|^2dxdydt \\ &\quad -\sum_{j=1}^m\sum_{i=1}^n\int_\Omega 4s\gamma^2\varphi\left(\nabla_y\psi\cdot\nabla_y z\right)\left(\nabla_x\psi\cdot\nabla_x\overline{z}\right)dxdydt. \end{aligned} \tag{3.23}$$

Before estimating J_3, we can directly verify

$$\begin{aligned} \Delta_y\left(\varphi d_2(\psi)\right) &= \left(\Delta_y\varphi\right)d_2(\psi) + 2\nabla_y\varphi\cdot\nabla_y\left(d_2(\psi)\right) + \varphi\Delta_y\left(d_2(\psi)\right) \\ &= \gamma\varphi\left(\Delta_y\psi\right)d_2(\psi) + \gamma^2\varphi\left|\nabla_y\psi\right|^2 d_2\psi + 2\gamma\varphi\nabla_y\psi\cdot\nabla_y\left(d_2(\psi)\right) \\ &\quad +\varphi\Delta_y\left(d_2(\psi)\right), \\ \Delta_y\left(\varphi d_1(\psi)\right) &= \left(\Delta_y\varphi\right)d_1(\psi) + 2\nabla_y\varphi\cdot\nabla_y\left(d_1(\psi)\right) + \varphi\Delta_y\left(d_1(\psi)\right) \\ &= \gamma\varphi\left(\Delta_y\psi\right)d_1(\psi) + \gamma^2\varphi\left|\nabla_y\psi\right|^2 d_1\psi + 2\gamma\varphi\nabla_y\psi\cdot\nabla_y\left(d_1(\psi)\right) \\ &\quad +\varphi\Delta_y\left(d_1(\psi)\right). \end{aligned}$$